

## 变截面波形钢腹板悬臂梁抗剪性能研究

王方旭<sup>1</sup>, 刘世忠<sup>1†</sup>, 杨霞林<sup>1</sup>, 秦翩翩<sup>2</sup>, 刘欣益<sup>1</sup>

(1. 兰州交通大学 土木工程学院, 甘肃 兰州 730070;

2. 枣庄学院 城市与建筑工程学院, 山东 枣庄 277160)

**摘要:**为了在变截面波形钢腹板悬臂梁的剪应力计算时考虑翼板、腹板以及组合箱梁截面转角差异,首先从翼板、腹板以及组合箱梁截面转角出发建立位移函数,采用能量变分法将翼板与腹板承剪进行剥离,利用弯矩等效分离顶、底板承剪;其次结合变截面波形钢腹板组合箱梁的梁段分析单元刚度矩阵及节点荷载列阵建立了剪应力求解程序;最后分析了悬臂梁在不同荷载工况下顶、底板及腹板剪应力与承剪比。结果表明:相较于有限元计算结果,考虑转角差异后顶板、腹板和底板承剪比计算结果较已有变截面剪应力计算结果精度最大可提高3.48%、3.43%和6.91%;悬臂梁各组件承剪比取决于荷载形式,梁端集中荷载作用下,顶、底板各自承剪比达到最大,分别为自由端的12.82%和固定端的60.81%;均布荷载作用下,腹板承剪比达到最大,为自由端的78.11%。

**关键词:**剪应力;能量变分法;有限梁段法;承剪比

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## Research on Shear Performance of Cantilever Beam with Variable-section Corrugated Steel Web

WANG Fangxu<sup>1</sup>, LIU Shizhong<sup>1†</sup>, YANG Xialin<sup>1</sup>, QIN Aoao<sup>2</sup>, LIU Xinyi<sup>1</sup>

(1. School of Civil Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China;

2. School of City and Architecture Engineering, Zaozhuang University, Zaozhuang 277160, China)

**Abstract:** To consider the difference in the section rotation angle of the wing flange, web, and composite box girder when calculating the shear stress of a cantilever beam with variable section corrugated steel web, a displacement function was first established based on the rotation angles of the flanges, web, and composite box girder. The energy variational method was used to separate the shear contribution of the flange and web, and the bending moment equivalent was used to further separate the top and bottom flange shear. Secondly, the shear-stress solving program was established based on the stiffness matrix of the beam segment analysis element and the joint load array of the composite box girder with variable section corrugated steel web. Lastly, the study analyzed the shear

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作者简介:王方旭(1988—),男,甘肃天水人,兰州交通大学博士

† 通信联系人,E-mail: 645819349@qq.com

stress and shear capacity ratio of the top flange, bottom flange, and steel web of the cantilever beam under various loading conditions. The results show that, when compared to the finite element calculation results, the accuracy of the calculation results of the shear capacity ratio of the top flange, web, and bottom flange after considering the difference of rotation angle can be improved by a maximum of 3.48%, 3.43%, and 6.91%, respectively, compared with the existing results of shear stress calculation of variable cross-section. The shear capacity ratio of each component of the cantilever beam depends on the load form. The values of the top flange and bottom flange reach the maximum under the concentrated load of the beam free-end, which is 12.82% of the free end and 60.81% of the fixed end, respectively, and the web shear capacity ratio reaches the maximum under the action of uniform load, which is 78.11% of the free end.

**Key words:** shear stress; energy variational method; finite beam segment method; shear capacity ratio

波形钢腹板组合箱梁因其有效地解决了混凝土腹板开裂问题而被广泛使用<sup>[1-3]</sup>。但波形钢腹板手风琴效应及剪切模量折减易引起剪切变形、剪切屈曲等问题<sup>[4-6]</sup>,而变截面波形钢腹板组合箱梁的该类问题更为突出且应用范围更为广泛<sup>[7-10]</sup>。因此,研究变截面波形钢腹板组合箱梁抗剪性能对该类桥梁的应用推广具有十分重要的力学意义。

在波形钢腹板组合箱梁的发展过程中,国内外众多学者对波形钢腹板组合箱梁的抗剪性能展开了一系列的研究。抗剪强度方面Luo等<sup>[11]</sup>研究了变截面波形钢腹板梁极限强度;Yi等<sup>[12]</sup>研究了波形钢腹板通用经典板屈曲;Papangelis等<sup>[13]</sup>研究了变截面波形钢腹板梁的抗剪承载力设计;Hassanein等<sup>[14-16]</sup>研究了波形钢腹板梁强度设计。剪应力计算方面Moon等<sup>[17]</sup>通过公式代换将波形钢腹板整体屈曲荷载及合成剪切屈曲应力计算公式发展成了一般的经典板屈曲计算公式;Nie等<sup>[18]</sup>基于线弹性屈曲分析进行了广泛的参数研究,得出用于计算考虑三种不同剪切屈曲模式的波形钢腹板弹性剪切屈曲强度的简化公式;Zhou等<sup>[7-10]</sup>在不考虑波形钢腹板翼板界面滑移的前提下考虑附加剪力,求得变截面波形钢腹板组合箱梁剪应力;Ye等<sup>[19]</sup>对超高性能混凝土-波形钢腹板预应力变截面组合箱梁的剪应力计算公式进行推导;与此同时,张玉元等<sup>[20]</sup>推导了利用各内力分量引起的剪应力分量叠加求解变截面薄壁箱梁剪应力的计算公式。承剪比方面Kadotani等<sup>[21]</sup>通过试验研究得出简支梁65%的剪力由波形钢腹板承担;Shitou等<sup>[22]</sup>指出翼板开裂导致波形钢腹板承剪比增大,但屈服后这一比例减小;Kato等<sup>[23]</sup>考虑了波形钢腹板与翼板之间的剪力分布,提出了波形钢腹板组合箱

梁的设计方法。虽然已有文献对剪应力计算开展了比较深入的研究,但经典等截面剪应力计算对于腹板剪应力计算过于保守,变截面剪应力计算未对顶、底板承剪进行剥离,使得顶底板承剪无法量化分析;同时已有文献推导变截面薄壁箱梁及变截面波形钢腹板组合箱梁剪应力计算公式均通过求解各内力分量引起的剪应力分量并叠加而得,且波形钢腹板组合箱梁手风琴效应会使翼板、腹板以及组合箱梁截面转角产生差异,因此亟须一种能体现波形钢腹板组合箱梁特有手风琴效应的剪应力计算方法。

本文从翼板、腹板以及组合箱梁截面转角出发建立位移函数,基于能量变分法及有限梁段法结合弯矩等效,将顶板、底板及腹板承剪进行剥离并求解,在此基础上,分析悬臂梁在不同荷载工况下顶板、底板、钢腹板剪应力及承剪比。

## 1 传统剪应力计算公式

### 1.1 等截面箱梁剪应力计算

假设剪力全部由腹板承担且剪应力沿高度方向均匀分布时:

$$\tau = \frac{Q}{A_w} \quad (1)$$

式中: $\tau$ 为腹板竖向剪应力; $Q$ 为全截面竖向剪力; $A_w$ 为腹板承剪面积。

### 1.2 变截面箱梁剪应力计算

参考文献[7,20],取微段 $dx$ 为研究对象,如图1所示,利用微元体平衡、剪应力互等定理可得变截面组合箱梁任意点的剪应力计算公式为:

$$\begin{aligned}\tau = & \frac{QS_a}{Ib_3} + \frac{N}{b_3} \left( \frac{S_a \tan \phi}{I} + \frac{1}{A} \frac{dA_a}{dx} - \frac{A_a}{A^2} \frac{dA}{dx} \right) + \\ & \frac{M}{Ib_3} \left( \frac{dS_a}{dx} - \frac{S_a}{I} \frac{dI}{dx} \right) = \tau_Q + \tau_N + \tau_M\end{aligned}\quad (2)$$

式中: $Q$ 、 $N$ 、 $M$ 分别表示全截面剪力、轴力和弯矩;  $\phi$ 为截面形心连线的水平倾角;  $b_3$ 为剪应力计算点的截面宽度;  $A_a$ 为剪应力计算点至梁顶的截面面积;  $A$ 、 $I$ 分别为横截面面积及对形心轴的惯性矩;  $S_a$ 为  $A_a$  对全截面形心轴的静矩;  $\tau_Q$ 、 $\tau_N$ 、 $\tau_M$  分别为由剪力、轴力、弯矩引起的剪应力分量.

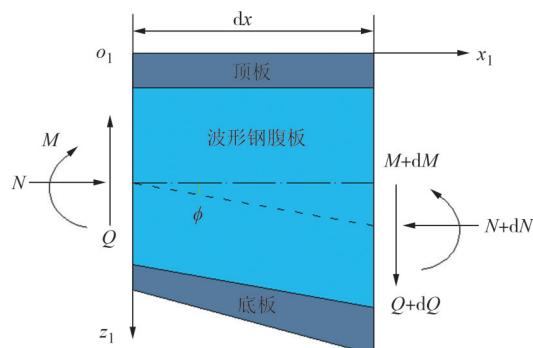


图 1 微段平衡示意图

Fig.1 Micro segment equilibrium diagram

式(2)存在  $\frac{dA_a}{dx}$ 、 $\frac{dA}{dx}$ 、 $\frac{dS_a}{dx}$ 、 $\frac{dI}{dx}$  四项微分项,且随着计算位置的不同,  $A_a$  与  $S_a$  表达式也不相同,则可计算不同位置处剪应力.

## 2 顶板、底板、腹板剪力分离及计算

### 2.1 基本假定

- 1) 结构处于弹性工作阶段,材料受力符合胡克定律;
- 2) 弹性工作阶段拟平截面假定成立;
- 3) 考虑波形钢腹板的手风琴效应,忽略波形钢腹板上的法向应变,并认为混凝土顶、底板主要贡献抗弯强度,波形钢腹板仅提供抗剪强度;
- 4) 忽略钢腹板混凝土翼板连接界面处的剪切滑移;
- 5) 剪应力沿高度方向均匀分布,不考虑顶、底板的有效分布宽度及剪切变形.

### 2.2 组合箱梁位移函数

波形钢腹板组合箱梁截面示意如图 2 所示. 图中坐标原点  $o$  位于截面形心处,  $y$ 、 $z$  为横截面的形心主惯性轴,  $b_1$ 、 $b_2$  分别表示顶板、底板的 1/2 宽度,  $h_s$ 、 $h_x$

分别表示截面形心到顶板底面、底板顶面的距离,  $h_w$  表示顶、底板间净距,  $z_s$ 、 $z_x$  分别表示截面形心到顶、底板中心的距离;  $t_1$ 、 $t_2$  分别为顶板、底板厚度.

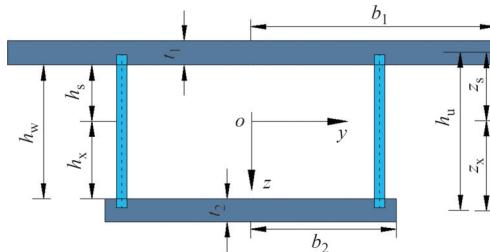


图 2 组合箱梁示意图

Fig.2 Composite box beam diagram

波形钢腹板组合箱梁承受任意荷载,考虑腹板手风琴效应,各组件变形如图 3 所示. 图中:  $\alpha(x)$  为顶、底板中心连线与  $z$  轴的夹角,  $w'(x)$  为顶、底板竖向挠曲转角,  $\theta(x)$  为波形钢腹板挠曲转角,  $u$ 、 $w$  分别表示组合箱梁的纵向和竖向位移.

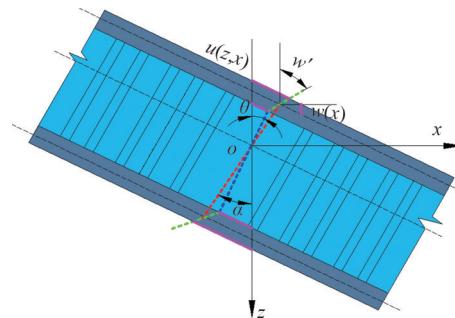


图 3 波形钢腹板变形模式示意图

Fig.3 Deformation mode diagram of corrugated steel web

由图 2 和图 3 可得,上下翼板的纵向位移为:

$$u_s(z, x) = z_s \alpha(x) - (z + z_s) w'(x) \quad (3)$$

$$u_x(z, x) = -z_x \alpha(x) - (z - z_x) w'(x) \quad (4)$$

由式(3)、式(4)可得:

$$\begin{aligned}\theta(z) = & \frac{u_s(-h_s, x) - u_x(h_s, x)}{h_s + h_x} = \\ & \lambda \alpha(x) + (1 - \lambda) w'(x)\end{aligned}\quad (5)$$

式中:  $\lambda = (z_s + z_x)/(h_s + h_x) = h_u/h_w$ .

由式(3)~式(5)可得腹板任意点的纵向位移为:

$$\begin{aligned}u_w(z, x) = & u_s(-h_s, x) - (z + h_s) \theta(x) = \\ & [z_s - \lambda(z + h_s)] \alpha(x) - \\ & [(z_s - h_s) - \chi(z + h_s)] w'(x)\end{aligned}\quad (6)$$

式中:  $\chi = \lambda - 1$ .

### 2.3 控制微分方程

由式(3)~式(6)可得:

顶板正应变为:

$$\varepsilon_x^t = z_s \alpha'(x) - (z + z_s) w''(x) \quad (7)$$

底板正应变为:

$$\varepsilon_x^b = -z_x \alpha'(x) - (z - z_s) w''(x) \quad (8)$$

腹板剪应变为:

$$\gamma_w = \lambda [w'(x) - \alpha(x)] \quad (9)$$

应变能为:

$$U = \frac{1}{2} \int_l \int_A \{ E_c [(\varepsilon_x^t)^2 + (\varepsilon_x^b)^2] + G_w (\gamma_w)^2 \} dA dx = \frac{1}{2} E_c \int_l (I_{y_0} \theta'^2 + I_{y_c} w''^2) dx + \frac{1}{2} G_w \int_l \lambda^2 A_w (w' - \alpha)^2 dx \quad (10)$$

其中:

$$I_{y_0} = z_s^2 A_s + z_x^2 A_x \quad (11)$$

$$I_{y_c} = \frac{1}{12} t_1^2 A_s + \frac{1}{12} t_2^2 A_x \quad (12)$$

$$G_w = \frac{a+b}{a+c} G_s, \quad A_w = h_w t_w \quad (13)$$

式中: $A_s$ 、 $A_x$ 和 $A_w$ 分别为组合箱梁顶板、底板和腹板的面积;  $E_c$ 为翼板混凝土弹性模量;  $G_s$ 为钢材的剪切模量;  $G_w$ 表示组合箱梁钢腹板的换算剪切模量;  $a$ 、 $b$ 、 $c$ 为波形参数,如图4所示.

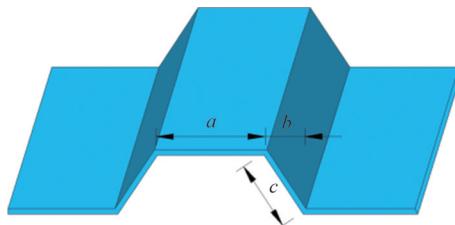


图4 波形参数示意图

Fig.4 Schematic diagram of waveform parameters

总势能为:

$$\Pi = \frac{1}{2} E_c \int_l (I_{y_0} \alpha'^2 + I_{y_c} w''^2) dx + \frac{1}{2} G_w \int_l \lambda^2 A_w (w' - \alpha)^2 dx - \int_0^l q(x) w(x) dx \quad (14)$$

由最小势能原理 $\delta\Pi=0$ ,可得:

$$\begin{aligned} \delta\Pi = & - \int_0^l [E_c I_{y_0} \alpha'' + \lambda^2 G_w A_w (w' - \alpha)] \delta\alpha dx + \\ & \int_0^l [E_c I_{y_c} w''' - \lambda^2 G_w A_w (w'' - \alpha') - q] \delta w dx + \\ & E_c I_{y_0} \alpha' \delta\alpha \Big|_0^l + E_c I_{y_c} w'' \delta w \Big|_0^l + \\ & [\lambda^2 G_w A_w (w' - \alpha) - E_c I_{y_c} w'''] \delta w \Big|_0^l \end{aligned} \quad (15)$$

控制微分方程为:

$$E_c I_{y_c} w''' - \lambda^2 G_w A_w (w'' - \alpha') = q \quad (16)$$

$$E_c I_{y_0} \alpha'' + \lambda^2 G_w A_w (w' - \alpha) = 0 \quad (17)$$

边界条件为:

$$E_c I_{y_c} w'' \delta w \Big|_0^l = 0 \quad (18)$$

$$E_c I_{y_0} \alpha' \delta\alpha \Big|_0^l = 0 \quad (19)$$

$$[\lambda^2 G_w A_w (w' - \alpha) - E_c I_{y_c} w'''] \delta w \Big|_0^l = 0 \quad (20)$$

由边界条件可知,顶板和底板变形产生的弯矩为:

$$M_\alpha = E_c I_{y_0} \alpha' \quad (21)$$

顶板和底板绕自身中心的弯矩为:

$$M_w = E_c I_{y_c} w'' \quad (22)$$

截面总剪力为:

$$Q = \lambda^2 G_w A_w (w' - \alpha) - E_c I_{y_c} w''' \quad (23)$$

其中,波形钢腹板承担的剪力为:

$$Q_w = \int_A G_w \gamma_w dA = \lambda G_w A_w (w' - \alpha) \quad (24)$$

顶板和底板承担的剪力为:

$$Q_c = \lambda \chi G_w A_w (w' - \alpha) - E_c I_{y_c} w''' \quad (25)$$

波形钢腹板剪应力为:

$$\tau_w = \frac{Q_w}{A_w} \quad (26)$$

求出 $w(x)$ 、 $\alpha(x)$ 、 $\theta(x)$ ,便可求解各剪力.

## 2.4 顶、底板承剪分离

如图5所示, $Q$ 为全截面剪力, $Q_{cs1}$ 、 $Q_{w1}$ 、 $Q_{cx1}$ 分别为顶板、腹板、底板本身所受剪力, $Q_{cs2}$ 、 $Q_{w2}$ 、 $Q_{cx2}$ 分别为顶板、腹板、底板附加剪力,且 $Q_{cs2}+Q_{w2}+Q_{cx2}=0$ .

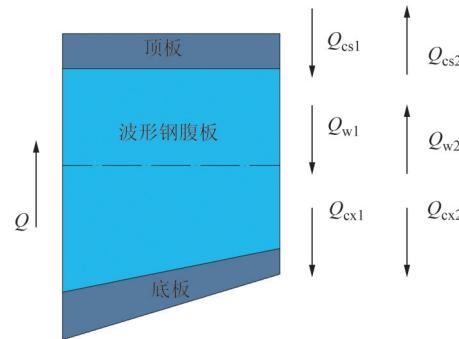


图5 箱梁受剪示意图

Fig.5 Shear diagram of box beam

$Q_{cs1}$ 、 $Q_{w1}$ 、 $Q_{cx1}$ 计算方法如下:

$$Q_{w1} = \frac{QS}{Ib_0} \cdot A_w \quad (27)$$

$$Q_{cs1} = (Q - Q_{w1}) \cdot \frac{A_s}{A_s + A_x} \quad (28)$$

$$Q_{cx1} = (Q - Q_{w1}) \cdot \frac{A_x}{A_s + A_x} \quad (29)$$

式中: $S$ 为横截面上该点以外部分的面积对中性轴的静距; $I$ 为横截面对中性轴的惯性矩; $b_0$ 为截面有效宽度.

参考文献[7],组合箱梁弯矩 $M$ 可以等效为一对作用在顶、底板截面形心上的平衡力系 $\{F_t, F_c\}$ ,如图6所示.

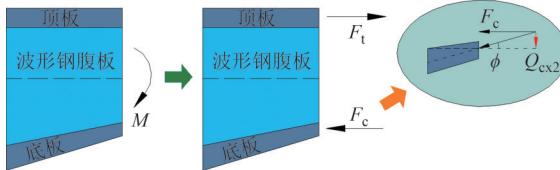


图6 等效弯矩示意图

Fig.6 Diagram of equivalent bending moment

则:

$$F_t = \frac{M}{0.5t_1 + h_w + 0.5t_2} = -F_c \quad (30)$$

$$Q_{ex2} = F_c \tan \varphi \quad (31)$$

$$Q_{ex} = Q_{ex1} + Q_{ex2} \quad (32)$$

$$Q_{cs} = Q - Q_w - Q_{ex} \quad (33)$$

式中: $\varphi$ 为底板形心轴与水平线的夹角; $Q_{ex}$ 为底板所受剪力; $Q_{cs}$ 为顶板所受剪力.

顶板剪应力为:

$$\tau_s = \frac{Q_{cs}}{A_s} \quad (34)$$

底板剪应力为:

$$\tau_x = \frac{Q_{ex}}{A_x} \quad (35)$$

## 2.5 承剪比

承剪比为:

$$\rho = \frac{Q_i}{Q} \quad (36)$$

式中: $Q_i$ 为各组件所受剪力.

## 3 转角位移求解

为求解 $w(x)$ 、 $\alpha(x)$ 、 $\theta(x)$ ,建立2节点6自由度的梁段单元,如图7所示.

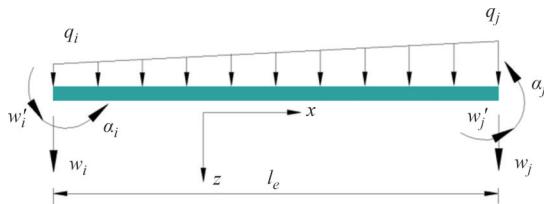


图7 有限梁段模型

Fig.7 Finite beam segment model

位移向量 $\delta^e$ 及其所对应的力向量 $F^e$ 为:

$$\delta^e = [w_i \ w'_i \ \alpha_i \ w_j \ w'_j \ \alpha_j]^T \quad (37)$$

$$F^e = [Q_i \ M_{wi} \ M_{ai} \ Q_j \ M_{wj} \ M_{aj}]^T \quad (38)$$

式中: $w_i$ 和 $w_j$ 分别表示梁段单元*i*端和*j*端的竖向位移; $w'_i$ 和 $w'_j$ 分别表示单元*i*端和*j*端顶、底板截面形心的转角; $\alpha_i$ 和 $\alpha_j$ 分别表示单元*i*端和*j*端顶、底板绕各板中心的转角; $Q_i$ 和 $Q_j$ 分别表示单元*i*端和*j*端截面的全部剪力; $M_{wi}$ 和 $M_{wj}$ 分别表示单元*i*端和*j*端顶、底板绕截面形心转动产生的弯矩; $M_{ai}$ 和 $M_{aj}$ 分别表示单元*i*端和*j*端顶、底板绕各自板中心转动产生的弯矩.

由式(16)和式(17)得:

$$w^{(6)} - k^2 w^{(4)} = \frac{q}{k_0} + \frac{q''}{E_c I_{yc}} \quad (39)$$

其中:

$$k^2 = \frac{\lambda^2 G_w A_w I_y}{E_c I_{yc} I_{y0}} \quad (40)$$

$$k_0 = \frac{E_c^2 I_{yc} I_{y0}}{\lambda^2 G_w A_w} \quad (41)$$

微分方程(39)的解为:

$$w(x) = C_1 + C_2 x + C_3 x^2 + C_4 x^3 + C_5 \cosh kx + C_6 \sinh kx + C_0 x^4 \quad (42)$$

式中: $C_0 x^4$ 为均布荷载作用时的特解, $C_1 \sim C_6$ 为积分常数.

由式(18)、式(19)得:

$$\frac{E_c I_{yc} I_{y0}}{\lambda^2 G_w A_w} w^{(4)} - E_c I_y w'' - \frac{q(x) E_c I_{y0}}{\lambda^2 G_w A_w} = M(x) \quad (43)$$

对无载梁段:

$$M(x) = -2E_c I_y C_3 - 6E_c I_y C_4 x \quad (44)$$

由式(20)可得:

$$\alpha(x) = w' - \frac{E_c I_y w''}{\lambda^2 G_w A_w} - \frac{Q}{\lambda^2 G_w A_w} \quad (45)$$

求解 $\alpha(x)$ 可得:

$$\alpha(x) = C_2 + 2C_3 x + C_4 (6\zeta + 3x^2) + C_5 s \sinh kx + C_6 s \cosh kx \quad (46)$$

其中:

$$\zeta = \frac{E_c I_{y0}}{\lambda^2 G_w A_w} \quad (47)$$

$$s = -k \frac{I_{yc}}{I_{y0}} \quad (48)$$

由式(39)~式(41)可得:

$$Q = M'(x) = 6E_c I_y C_4 \quad (49)$$

$$M_{wi} = -E_c I_{yc} (2C_3 + 6C_4 x +$$

$$k^2 C_5 \cosh kx + k^2 C_6 \sinh kx) \quad (50)$$

$$M_{\alpha i} = -E_c I_{y0} (2C_3 + 6C_4 x + k\zeta C_5 \cosh kx + k\zeta C_6 \sinh kx) \quad (51)$$

将式(37)改写为:

$$\delta^e = A_E \mathbf{C} \quad (52)$$

向量  $\mathbf{C}$  及  $A_E$  为:

$$\mathbf{C} = [C_1 \ C_2 \ C_3 \ C_4 \ C_5 \ C_6]^T \quad (53)$$

$$A_E = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & k \\ 0 & 1 & 0 & 6\zeta & 0 & \zeta \\ 1 & l & l^2 & l^3 & \cosh kl & \sinh kl \\ 0 & 1 & 2l & 3l^2 & k \sinh kl & k \cosh kl \\ 0 & 1 & 2l & 6\zeta + 3l^2 & s \sinh kl & s \cosh kl \end{bmatrix} \quad (54)$$

将式(38)改写为:

$$\mathbf{F}^e = B_E \mathbf{C} \quad (55)$$

$$B_E = \begin{bmatrix} 0 & 0 & 0 & 6E_c I_y & 0 & 0 \\ 0 & 0 & -2E_c I_{yc} & 0 & -k^2 E_c I_{yc} & 0 \\ 0 & 0 & -2E_c I_{y0} & 0 & -ks E_c I_{y0} & 0 \\ 0 & 0 & 0 & -6E_c I_y & 0 & 0 \\ 0 & 0 & 2E_c I_{yc} & 6E_c I_{yc} l & k^2 E_c I_{yc} \sinh kl & k^2 E_c I_{yc} \cosh kl \\ 0 & 0 & 2E_c I_{y0} & 6E_c I_{y0} l & ks E_c I_{y0} \sinh kl & ks E_c I_{y0} \cosh kl \end{bmatrix} \quad (56)$$

将系数矩阵代入式(52)可得:

$$\mathbf{F}^e = B_E A_E^{-1} \delta^e = K^e \delta^e \quad (57)$$

单元任一点处的位移为:

$$u^{\delta} = K_u A_E^{-1} \delta^e = N \delta^e = [w \ w' \ \alpha]^T \quad (58)$$

位移形函数  $N$  为:

$$N = \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15} & N_{16} \\ N_{21} & N_{22} & N_{23} & N_{24} & N_{25} & N_{26} \\ N_{31} & N_{32} & N_{33} & N_{34} & N_{35} & N_{36} \end{bmatrix} \quad (59)$$

$$K_u = \begin{bmatrix} 1 & x & x^2 & x^3 & \cosh kx & \sinh kx \\ 0 & 1 & 2x & 3x^2 & k \sinh kx & k \cosh kx \\ 0 & 1 & 2x & 6\zeta + 3x^2 & s \sinh kx & s \cosh kx \end{bmatrix} \quad (60)$$

根据虚功原理, 可求得均布荷载作用时的等效节点荷载为:

$$F^{eq} = \int_0^l q N^T dx = -q \int_0^l [N_{11} \ N_{12} \ N_{13} \ N_{14} \ N_{15} \ N_{16}]^T dx \quad (61)$$

$$F_q = [F_1^q \ F_2^q \ F_3^q \ F_4^q \ F_5^q \ F_6^q]^T \quad (62)$$

式中:

$$\begin{cases} F_1^q = F_4^q = \frac{ql}{2} \\ F_2^q = -F_5^q = \frac{ql^2 v}{12} \\ F_3^q = -F_6^q = \frac{ql^2(1-v)}{12} \end{cases} \quad (63)$$

$$v = \frac{6kl + skl^2 + 12}{(s-k)kl^2} + \frac{12e^{kl}}{(1-e^{kl})(s-k)l} \quad (64)$$

转角位移的求解, 采用计算软件编写程序进行.

## 4 算例分析

### 4.1 几何参数

参考文献[7]中计算模型, 其中悬臂梁的跨度30 m, 顶板宽4 m, 底板宽2 m, 梁高由固定端4 m线性变化至自由端2 m, 顶板厚度0.3 m, 底板厚由固定端0.5 m线性变化至自由端0.3 m, 波形钢腹板为1000型(CSW1000), 钢腹板厚度0.01 m. 如图8所示.

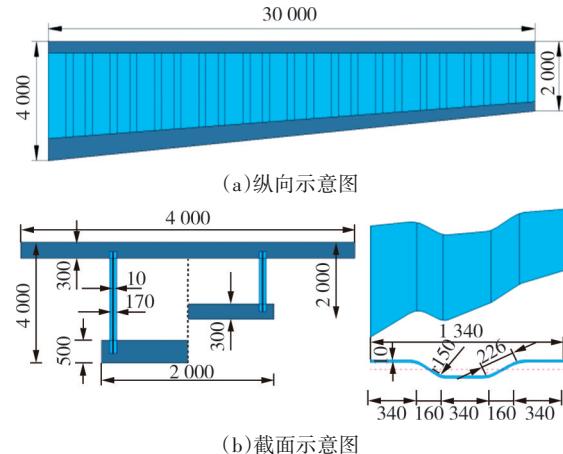


图8 变截面梁几何尺寸(单位:mm)

Fig.8 Geometric dimension of variable section beam (unit:mm)

### 4.2 有限元模型

根据模型尺寸, 采用ANSYS软件建立三维有限元分析模型, 如图9所示. 其中顶板、底板采用实体单元SOLID65模拟, 波形钢腹板采用壳单元SHELL63模拟, 顶板、底板混凝土材料弹性模量  $E=3.45 \times 10^4$  MPa, 泊松比  $\mu=0.2$ ; 腹板钢材弹性模量  $E=2.1 \times 10^5$  MPa, 泊松比  $\mu=0.3$ . 顶板、底板实体单元与腹板壳单元采用共节点连接, 固定端( $x=0$ )约束平动和转动自由度.

### 4.3 转角位移求解

采用MATLAB计算软件编写对应的梁段分析程

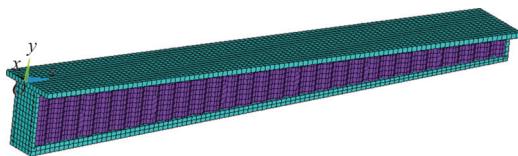


图9 有限元模型示意图  
Fig.9 Diagram of finite element model

序,进行转角位移的求解.模型梁划分30个单元,单元尺寸1 m,计算流程如图10所示.

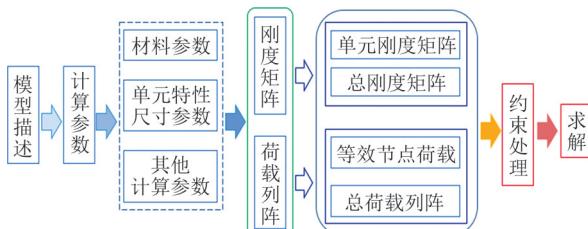


图10 计算流程图  
Fig.10 Calculation flowchart diagram

#### 4.4 剪应力分析

##### 4.4.1 均布荷载

在悬臂梁上施加均布荷载 $q=35 \text{ kN/m}$ ,采用对称加载,如图11所示.为减少边界效应带来的误差,将固定端1号截面和自由端2号截面向内侧移动1 m.同时,对式(2)、式(26)和有限元模型的计算结果进行对比,验证模型的可靠性.

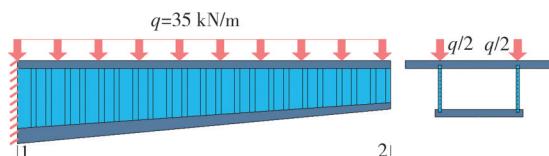


图11 均布荷载加载示意图  
Fig.11 Diagram of uniformly load

将本文推导的计算结果与式(1)、式(2)、有限元计算结果作对比,同时以有限元计算结果作为参考进行误差分析,分析结果如图12~图14所示.

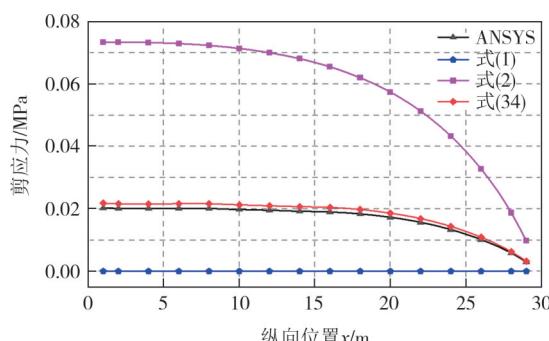


图12 均布荷载作用下的顶板剪应力  
Fig.12 Shear stress of top flange under uniform load

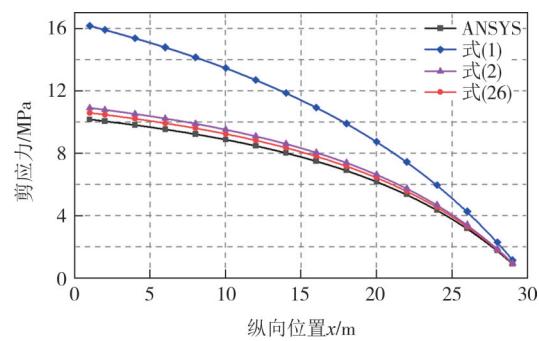


图13 均布荷载作用下的腹板剪应力  
Fig.13 Shear stress of web under uniform load

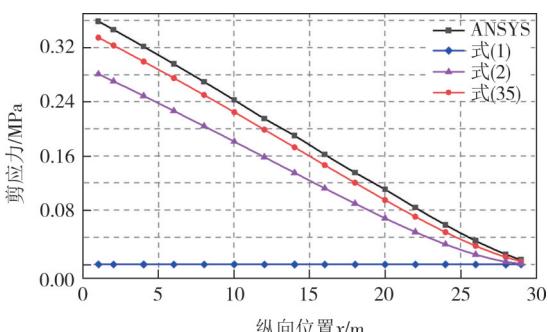


图14 均布荷载作用下的底板剪应力  
Fig.14 Shear stress of bottom flange under uniform load

由图12可知,本文式(34)计算结果介于式(1)与式(2)与有限元结果之间,且与有限元结果吻合良好,误差在6%以内;式(1)、式(2)结果与有限元结果差值较大,式(2)最大差值达到2倍以上甚至更大.

由图13可知,本文式(26)计算结果介于公式(2)与有限元结果之间,且与有限元结果的误差在5%以内.式(1)忽略翼板对竖向剪力的贡献,使得截面1处腹板剪应力达到了有限元结果的1.59倍,这与实际抗力组成存在较大差异,随着跨度变化,其差值由截面1的58.95%减小到截面2的28.42%.截面1处式(2)与有限元结果差值为7.29%,式(26)为4.17%.计算结果中式(1)最大,其次为式(2)、式(26)、有限元.

由图14可知,本文式(35)计算结果与有限元以及式(2)结果变化趋势大致相同,且与有限元值吻合良好,误差在6%以内.

##### 4.4.2 集中荷载

在悬臂梁端部作用一竖向集中力 $P=500 \text{ kN}$ ,加载示意图如图15所示,计算结果如图16~图18所示.

由图16可知,本文式(34)计算结果介于式(1)与式(2)结果之间,且与有限元结果吻合良好,式(2)与有限元差值较大.

由图17可知,本文式(26)计算结果介于有限元

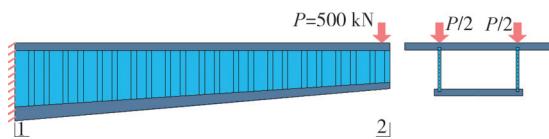


图 15 集中荷载加载示意图

Fig.15 Diagram of concentrated load

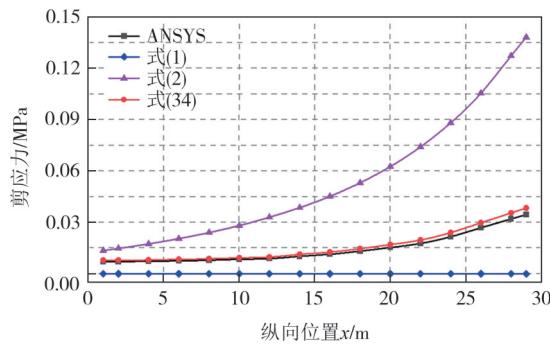


图 16 集中荷载作用下的顶板剪应力

Fig.16 Shear stress of top flange under concentrated load

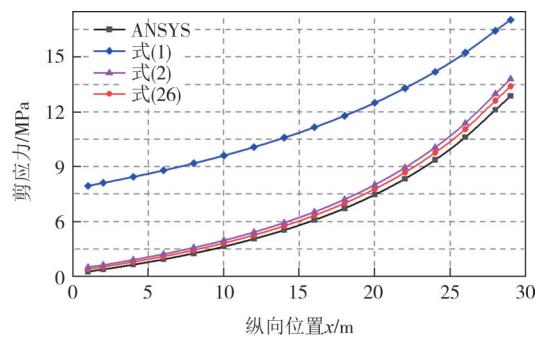


图 17 集中荷载作用下的腹板剪应力

Fig.17 Shear stress of web under concentrated load

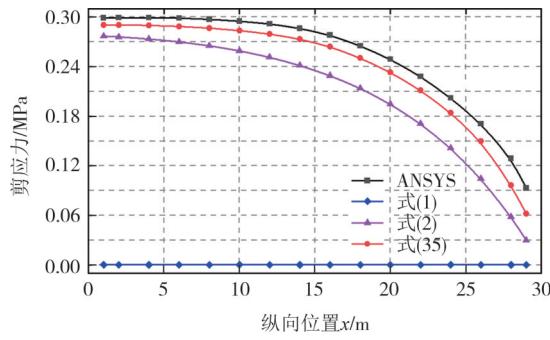


图 18 集中荷载作用下的底板剪应力

Fig.18 Shear stress of bottom flange under concentrated load

与式(2)结果之间,三者吻合良好,且与有限元结果误差在5%以内。计算结果中式(1)最大,其次为式(2)、式(26)、有限元。随着跨度的变化,式(1)与有限元差值与有限元计算结果比值由截面1处的2.44减小到截面2处的1.31。

由图18可知,本文式(35)计算结果与有限元结果以及式(2)结果变化趋势大致相同,且与有限元结

果吻合良好,误差在6%以内。

#### 4.5 承剪比分析

为了更直观地表达各组件的承剪情况,用承剪比来进一步说明各组件对抗剪的贡献,选取4种荷载工况(工况I  $q=35 \text{ kN/m}$ ,工况II  $P=500 \text{ kN}$ ,工况III  $q=17 \text{ kN/m}$ ,工况IV  $P=300 \text{ kN}$ ),其中工况I与工况II在截面1处弯矩近似相等;工况II与工况III在截面1处剪力近似相等;工况I与工况III荷载形式相同,大小不同;工况II与工况IV荷载形式相同,大小不同。利用式(36)求得顶、底板、波形钢腹板的承剪比。不同工况下顶、底板和波形钢腹板承剪比如图19~图21及表1所示。

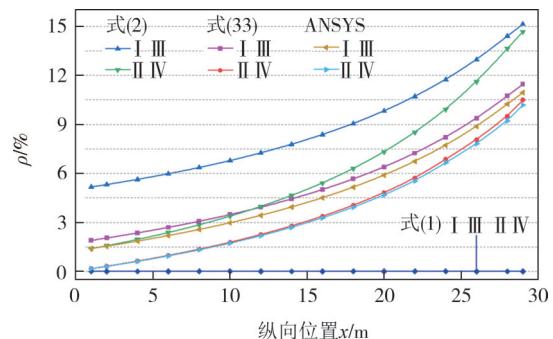


图 19 顶板承剪比

Fig.19 Shear capacity ratio of top flange

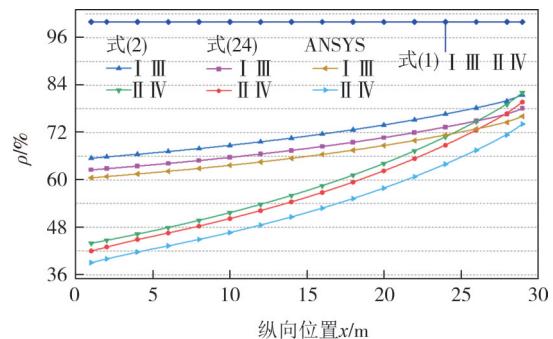


图 20 腹板承剪比

Fig.20 Shear capacity ratio of web

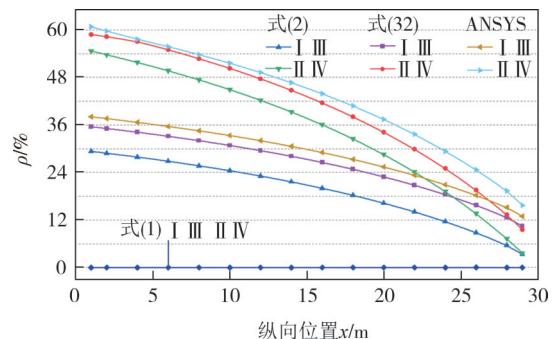


图 21 底板承剪比

Fig.21 Shear capacity ratio of bottom flange

表1 工况 I~IV 承剪比  
Tab.1 Condition I~IV shear capacity ratio

工况	承剪比/%								
	式(1)		式(2)		ANSYS		式(33)/(24)/(32)		
	截面1	截面2	截面1	截面2	截面1	截面2	截面1	截面2	
顶板	I	0	0	5.16	14.63	1.41	10.95	1.90	11.15
	II	0	0	1.39	14.25	0.16	10.19	0.12	12.82
	III	0	0	5.16	14.63	1.41	10.95	1.90	11.15
	IV	0	0	1.39	14.25	0.16	10.19	0.12	12.82
腹板	I	100	100	65.49	81.54	60.53	76.12	62.53	78.11
	II	100	100	41.94	80.10	39.06	74.12	39.07	76.71
	III	100	100	65.49	81.54	60.53	76.12	62.53	78.11
	IV	100	100	41.94	80.10	39.06	74.12	39.07	76.71
底板	I	0	0	29.35	3.83	38.06	12.93	35.57	10.74
	II	0	0	56.67	5.65	60.78	15.69	60.81	10.47
	III	0	0	29.35	3.83	38.06	12.93	35.57	10.74
	IV	0	0	56.67	5.65	60.78	15.69	60.81	10.47

由图19~图21、表1及本文计算公式可知：

1) 工况I、Ⅲ各截面承剪比相同,工况II、Ⅳ各截面承剪比相同,但工况I、Ⅲ与II、Ⅳ不同,由此说明组合悬臂梁承剪比取决于荷载形式.

2) 随着跨度增加,顶板和腹板的承剪比逐渐增大而底板的承剪比逐渐减小. 梁端集中荷载作用下,顶、底板各自承剪比达到最大,分别为自由端的12.82%和固定端的60.81%; 均布荷载作用下,腹板承剪比达到最大,为自由端的78.11%.

3) 荷载作用下,顶板承剪比与式(2)结果最大差值为工况I、Ⅲ下截面2处的3.48%,腹板最大差值为工况I、Ⅲ下截面2处3.43%,底板最大差值为工况I、Ⅲ下截面2处6.91%. 结合有限元模型,本文计算方法可使顶板计算精度最大提高3.48%,腹板计算精度最大提高3.43%,底板计算精度最大提高6.91%.

## 5 结论

本文从翼板、腹板以及组合箱梁截面转角差异出发,求解变截面波形钢腹板悬臂梁顶板、底板、腹板剪应力及承剪比,结合算例分析得出如下结论:

1) 采用等截面剪应力计算公式高估腹板承剪的最大误差达到2.44倍.

2) 变截面波形钢腹板悬臂梁由固定端到自由端,顶板、腹板承剪比逐渐增大,底板承剪比逐步减小.

3) 变截面波形钢腹板悬臂梁顶板、底板及腹板承剪比取决于荷载形式; 梁端集中荷载作用下,顶、底板各自承剪比达到最大,分别为自由端的12.82%和固定端的60.81%; 均布荷载作用下,腹板承剪比达到最大,为自由端的78.11%.

4) 从截面转角出发可使均布荷载下顶板、腹板和底板承剪比的计算精度最多提高3.48%、3.43%和6.91%.

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