第 49 卷 第 3 期 湖 南 大 学 学 报（自 然 科 学 版） Vol.49，No.3
2 0 2 2 年 3 月 Journal of Hunan University（Natural Sciences） Mar. 2 0 2 2

文章编号：1674-2974（2022）03-0123-10 DOI：10.16339/j.cnki.hdxbzkb.2022033

基于离散加筋理论的纵环加筋圆柱壳轴压后屈曲缩尺模型研究

李正良 [1,2]，王静超 [1]，于伟 [3,4†]，朱万旭 [3,4]
（1. 重庆大学 土木工程学院，重庆 400045；
2. 山地城镇建设与新技术教育部重点实验室（重庆大学），重庆 400045；
3. 桂林理工大学 土木与建筑工程学院，广西 桂林 541004；
4. 桂林理工大学 广西建筑新能源与节能重点实验室，广西 桂林 541004）

摘 要：针对新型干式煤气柜柜体（大型薄壁纵环离散加筋圆柱壳）轴压后屈曲的缩尺模型设计和相似预报问题，基于离散加筋理论与结构体系的总能量推导出结构轴压后屈曲的广义相似条件和缩尺原理公式. 对含不同初始缺陷类型和模型材料的纵环加筋圆柱壳，开展轴压后屈曲不完全相似模拟研究. 研究表明：纵环加筋圆柱壳缩尺模型的轴压后屈曲特性，结合结构轴压后屈曲不完全相似的相应缩尺原理公式，能较好地预测其原型结构轴压后屈曲的结果. 当模型与原型的材料泊松比相同时，模型能准确预测其原型的轴压后屈曲特性. 随着模型与原型的材料泊松比偏差增大，预测原型的后屈曲特性与其原型结果的偏差逐渐增大. 提出的缩尺模型设计方法和缩尺原理公式，适用于不同初始缺陷类型和模型材料的纵环加筋圆柱壳轴压后屈曲相似预报，为实际工程中类似结构轴压后屈曲缩尺模型设计和试验提供参考.

关键词：薄壁结构；气柜；离散加筋理论；后屈曲；缩尺模型
中图分类号：TU 33 **文献标志码**：A

Study on Post-buckling Scale Model of Stringer Stiffened-cylindrical Shells under Axial Compression Based on Discrete Stiffened Method

LI Zhengliang[1,2]，WANG Jingchao[1]，YU Wei[3,4†]，ZHU Wanxu[3,4]
（1. School of Civil Engineering，Chongqing University，Chongqing 400045，China；
2. Key Laboratory of New Technology for Construction of Cities in Mountain Area（Chongqing University）of the Ministry of Education，Chongqing 400045，China；
3. College of Civil and Architecture Engineering，Guilin University of Technology，Guilin 541004，China；
4. Guangxi Key Laboratory of New Energy and Building Energy Saving，Guilin University of Technology，Guilin 541004，China）

Abstract：Aiming at the scale model design and similarity prediction of the buckling of the new typical gas holder's body (large thin-walled ring and stringer stiffened-cylindrical shell) under axial compression, general si-

* **收稿日期**：2021-01-14
基金项目：国家自然科学基金资助项目（51868013），National Natural Science Foundation of China（51868013）；广西自然科学基金资助项目（2018GXNSFBA138056），Natural Science Foundation of Guangxi Province（2018GXNSFBA138056）；广西科技重大专项资助项目（桂科AA18118008），Guangxi Innovation-driven Development Special Project（GuiKe AA18118008）；桂林理工大学博士科研启动基金（GUTQDJJ2017034），Doctoral Research Initiation Fund of Guilin University of Technology（GUTQDJJ2017034）.
作者简介：李正良（1963—），男，江苏江阴人，重庆大学教授，博士生导师
† 通信联系人，E-mail：yuwei3601@163.com

militude requirements and scaling laws of stiffened shells under axial compression are presented based on the discrete stiffened method and the total energy of the structural system. For the ring and stringer stiffened-cylindrical shells with different initial imperfections and modeling material properties, the partial similitude of the post-buckling behaviors under axial compression is mainly investigated. The results show that: using partial similitude scaling laws for the post-buckling of stiffened cylindrical shells under axial compression, scaled models can accurately predict the post-buckling behaviors of prototypes of discretely ring and stringer stiffened-cylindrical shells under axial compression. When the material Poisson's ratio of the model and the prototype is the same, the model can accurately predict the post-buckling behaviors of the prototype under axial compression. As the material Poisson's ratio deviation between the model and the prototype increases, the deviation of the post-buckling behaviors between the predicted prototype and its corresponding prototype gradually increases. In conclusion, the design method of scale models and scale laws are suitable for the similarity prediction of the post-buckling behaviors of discretely ring and stringer stiffened-cylindrical shells with different initial imperfections and material properties, which provides a reference for the scale model design and test of similar structures in practical engineering.

Key words: thin-walled structures; gas holder; the discrete stiffened method; post-buckling; scaled model

新型干式煤气柜是一种可储存可燃气体,是能节约能源和保护环境的大型重要构筑物[1]. 新型干式煤气柜柜体作为一种典型的大型薄壁纵环离散加筋圆柱壳,由于试验条件和经费的限制,很难对其进行大量的原型结构试验. 目前,较为可行的研究方法是采用缩尺模型的试验结果结合相似理论预测原型的屈曲承载能力. 因此,研究纵环加筋圆柱壳轴压后屈曲缩尺模型的设计方法和相似原理,对验证与评估新型干式煤气柜柜体合理性和安全性显得尤为重要.

杨金花等[2]研究了具有环向贯穿脱层圆柱壳的屈曲问题,讨论了脱层大小、深度、位置以及复合材料纤维铺层方向对脱层圆柱壳屈曲载荷的影响. 结果表明:脱层长度越大、越靠近壳的外表和轴向中心,结构的屈曲载荷越低. 向红等[3]根据 Von Karman 板理论,建立了具损伤正交各向异性板的非线性压曲方程. Singhatanadgid 和 Ungbhakorn[4]针对压扭组合荷载作用下的正交各向异性圆板,进行了线性屈曲完全相似和不完全相似研究. 结果表明:完全相似模型和采用各向同性材料的不完全相似模型,结合缩尺原理公式,可以较好地预测其原型的屈曲特性,缩尺原理公式适用于具有任意相同边界条件的一对模型和原型. Hilburger 等[5-6]考虑加筋圆柱壳焊接缺陷的影响,根据结构的半径与等效厚度比值相等的原则,采用密加筋理论、有限元法和结构试验相结合的手段,对缩尺模型与原型进行了稳定性设计和试验,验证了新方法的正确性;同时给出了加筋圆柱壳模型详细和实用的设计方法,但尚未给出缩尺模型与原型的屈曲相似关系. 贾冬云等[7-8]采用理论分析、有限元模拟和试验研究相结合的方法,对大型正多边形煤气柜立柱、加筋肋和壁板进行了系统的研究. 总体而言,现有研究可以解决层合壳、板和密加筋圆柱壳屈曲的缩尺模型设计与相似预报问题,但对于纵环离散加筋圆柱壳屈曲相似的研究颇为缺乏,所以开展基于离散加筋理论的纵环加筋圆柱壳轴压后屈曲缩尺模型研究非常有必要.

本文以纵环加筋圆柱壳为研究对象,基于离散加筋理论与结构体系的总能量,推导出结构轴压后屈曲的广义相似条件与缩尺原理公式. 基于酒窝缺陷和焊缝缺陷的函数,建立了含初始缺陷的纵环加筋圆柱壳有限元模型. 最后,对含初始缺陷的纵环加筋圆柱壳,开展轴压后屈曲不完全相似模拟研究,以验证缩尺模型设计的广义相似条件和轴压后屈曲缩尺原理公式的准确性.

1 基于离散加筋理论的加筋圆柱壳轴压后屈曲缩尺原理

1.1 纵环离散加筋圆柱壳轴压后屈曲的广义相似条件

在相似转换过程中,相似结构体系的数学模型是一致的. 因此,任意两相似结构体系总能量间的关系可表示为:

$$\Pi_{\mathrm{p}}\left(X_{\mathrm{p}i}\right)=\psi\left(C_i\right)\Pi_{\mathrm{m}}\left(X_{\mathrm{m}i}\right) \tag{1}$$

式中:下标 p 和 m 分别代表原型和模型;$X_{\mathrm{p}i}$ 和 $X_{\mathrm{m}i}$(i=1,2,…,n)分别对应原型结构和模型结构的几何参数和材料参数;$\psi(C_i)$(i=1,2,…,n)为两相似结构参数的传递函数.根据该函数关系可推导出结构的屈曲缩尺原理公式.

纵环离散加筋圆柱壳结构见图 1、图 2.

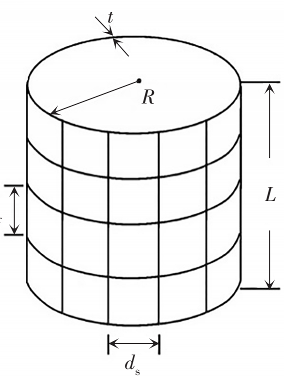
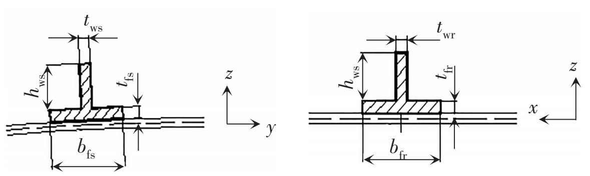


图 1　纵环离散加筋圆柱壳

Fig.1　The ring and stringer stiffened-cylindrical shell



(a)纵向加筋肋　　(b)环向加筋肋

图 2　T型纵环离散加筋圆柱壳的加筋肋截面图

Fig.2　Configurations of the ring and stringer T-shaped stiffened-cylindrical shell

图中,R 为蒙皮半径,t 为蒙皮厚度,L 为加筋圆柱壳长度.下标 s 和 r 分别代表纵向和环向的加筋肋.d_{s}和d_{r}分别表示纵向和环向的加筋肋间距.b_{fs}、t_{fs}和h_{ws}、t_{ws}分别为纵向T型加筋肋翼缘和腹板的长度与厚度.b_{fr}、t_{fr}和h_{wr}、t_{wr}分别为环向T型加筋肋翼缘和腹板的长度与厚度.

加筋圆柱壳的能量泛函为[9-10]:

$$\Pi = U + U_{\mathrm{L}} \tag{2}$$

式中:U、U_{L}分别为结构的应变能和外力功.

蒙皮的内力表达式为[10]:

$$\begin{gathered} N_x = B\left(\varepsilon_x+\mu\varepsilon_y\right),\ N_y = B\left(\varepsilon_y+\mu\varepsilon_x\right)\\ N_{xy} = Gt\gamma_{xy},\ M_x = D\left(\chi_x+\mu\chi_y\right)\\ M_y = D\left(\chi_y+\mu\chi_x\right),\ M_{xy} = D\left(1-\mu\right)\chi_{xy} \end{gathered} \tag{3}$$

式中:N_x、N_y和N_{xy}分别为单位长度上对应方向的薄膜内力;M_x、M_y和M_{xy}分别为单位长度上对应方向的弯矩;ε_x、ε_y和γ_{xy}为壳体蒙皮中面的应变;χ_x、χ_y和χ_{xy}是壳体曲率;μ为蒙皮材料泊松比;B、D和G分别为拉伸刚度、弯曲刚度和剪切模量,具体表达式为

$$B=\frac{Et}{1-\mu^2},D=\frac{Et^3}{12\left(1-\mu^2\right)},G=\frac{E}{2\left(1+\mu\right)} \tag{4}$$

基于Donnell假定,含初始几何缺陷圆柱壳的非线性几何方程为[11]:

$$\begin{gathered} \varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial \overline{w}}{\partial x}\\ \varepsilon_y=\frac{\partial v}{\partial y}+\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial \overline{w}}{\partial y}\\ \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial \overline{w}}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial \overline{w}}{\partial x}\\ \chi_x=-\frac{\partial^2 w}{\partial x^2},\chi_y=-\frac{\partial^2 w}{\partial y^2},\chi_{xy}=-\frac{\partial^2 w}{\partial x\partial y} \end{gathered} \tag{5}$$

式中:u、v、w分别为圆柱壳的纵向、环向和径向的位移;$\overline{w}$为径向初始几何缺陷.

加筋肋的非线性几何方程为[10]:

$$\varepsilon_{x_{\mathrm{s}}}=\varepsilon_x-e_{\mathrm{s}}\frac{\partial^2 w}{\partial x^2},\ \varepsilon_{y_{\mathrm{r}}}=\varepsilon_y-e_{\mathrm{r}}\frac{\partial^2 w}{\partial y^2} \tag{6}$$

式中:e为加筋肋形心到蒙皮中面的距离.

为了得到加筋圆柱壳的总能量的具体表达式,引入蒙皮与加筋肋的应变能表达式[10,12]:

$$\begin{gathered} U_0=\frac{1}{2}\int_0^{2\pi R}\int_0^L\Big(N_x\varepsilon_x+N_{xy}\gamma_{xy}+N_y\varepsilon_y-M_x\frac{\partial^2 w}{\partial x^2}-\\ 2M_{xy}\frac{\partial^2 w}{\partial x\partial y}-M_y\frac{\partial^2 w}{\partial y^2}\Big)\mathrm{d}x\mathrm{d}y\\ U_{\mathrm{s}}=\sum_{n_{\mathrm{s}}=1}^{N_{\mathrm{s}}}\left(\int_0^{2\pi R}\int_0^L\left(\begin{array}{l} E_{\mathrm{s}}A_{\mathrm{s}}\varepsilon_x^2\\ -2E_{\mathrm{s}}A_{\mathrm{s}}e_{\mathrm{s}}\varepsilon_x\dfrac{\partial^2 w}{\partial x^2}\\ +E_{\mathrm{s}}I_{0\mathrm{s}}\left(\dfrac{\partial^2 w}{\partial x^2}\right)^2\\ +G_{\mathrm{s}}J_{\mathrm{s}}\left(\dfrac{\partial^2 w}{\partial x\partial y}\right)^2 \end{array}\right)\delta(y-n_{\mathrm{s}}d_{\mathrm{s}})\mathrm{d}x\mathrm{d}y\right)\\ U_{\mathrm{r}}=\sum_{n_{\mathrm{r}}=1}^{N_{\mathrm{r}}}\left(\int_0^{2\pi R}\int_0^L\left(\begin{array}{l} E_{\mathrm{r}}A_{\mathrm{r}}\varepsilon_y^2\\ -2E_{\mathrm{r}}A_{\mathrm{r}}e_{\mathrm{r}}\varepsilon_y\dfrac{\partial^2 w}{\partial y^2}\\ +E_{\mathrm{r}}I_{0\mathrm{r}}\left(\dfrac{\partial^2 w}{\partial y^2}\right)^2\\ +G_{\mathrm{r}}J_{\mathrm{r}}\left(\dfrac{\partial^2 w}{\partial x\partial y}\right)^2 \end{array}\right)\delta(x-n_{\mathrm{r}}d_{\mathrm{r}})\mathrm{d}x\mathrm{d}y\right) \end{gathered} \tag{7}$$

式中:U_0、U_s和U_r分别为蒙皮、纵向加筋肋与环向加筋肋的应变能;e_s和e_r分别为纵肋截面形心与环肋截面形心到圆柱壳中面的距离;I_{0s}和I_{0r}分别为纵肋与环肋到圆柱壳中面的惯性矩;A为加筋肋截面面积;N为加筋肋数量;J_s和J_r为开口截面型加筋肋的扭转常量,其具体表达式为[13]

$$J = \frac{1}{3}\left(b_f t_f^3 + h_w t_w^3\right) \tag{8}$$

基于离散加筋理论的加筋圆柱壳应变能表达式为:

$$U = U_0 + U_s + U_r \tag{9}$$

轴压作用下结构的外力功表达式为[10]:

$$U_L = -\frac{1}{2}\int_0^{2\pi R}\int_0^L p_x\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x\mathrm{d}y - \frac{1}{2}\sum_{n_s=1}^{N_s}\int_0^L\left[\frac{p_x A_s}{t}\left(\frac{\partial w}{\partial x}\right)^2\right]\Bigg|_{y=n_s d_s}\mathrm{d}x \tag{10}$$

式中:p_x为壳体轴向均布荷载.

根据狄拉克函数的性质[14]:

$$\int_0^{2\pi R} f(y)\delta(y-n_s d_s)\mathrm{d}y = f(y)|_{y=n_s d_s},\ 0 \leqslant y \leqslant 2\pi R \tag{11}$$

式(10)可以转化为:

$$U_L = -\frac{1}{2}\int_0^{2\pi R}\int_0^L p_x\left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x\mathrm{d}y - \frac{1}{2}\int_0^{2\pi R}\int_0^L \frac{p_x A_s}{t}\left(\frac{\partial w}{\partial x}\right)^2\delta(y-n_s d_s)\mathrm{d}x\mathrm{d}y \tag{12}$$

首先,将式(3)、式(5)和式(6)代入式(7),推得式(9)的具体表达式.然后,将式(9)和式(12)代入式(2),得到加筋圆柱壳的总能量表达式为:

$$\begin{aligned}
\Pi = \frac{1}{2}\int_0^{2\pi R}\int_0^L\Bigg\{ & B\left\{\mu\left[\frac{\partial v}{\partial y}+\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial \bar{w}}{\partial y}\right]+\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial \bar{w}}{\partial x}\right\}\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial \bar{w}}{\partial x}\right]+ \\
& B\left\{\mu\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial \bar{w}}{\partial x}\right]+\frac{\partial v}{\partial y}+\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial \bar{w}}{\partial y}\right\}\left[\frac{\partial v}{\partial y}+\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial \bar{w}}{\partial y}\right]+ \\
& D\left[\mu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right]+D\left[\mu\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}+\left(\frac{\partial^2 w}{\partial y^2}\right)^2\right]+ \\
& Gt\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial \bar{w}}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial \bar{w}}{\partial x}\right)^2+2D(1-\mu)\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\Bigg\}\mathrm{d}x\mathrm{d}y+ \\
& \sum_{n_s=1}^{N_s}\int_0^{2\pi R}\int_0^L\left(\begin{aligned}& E_s A_s\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial \bar{w}}{\partial x}\right)^2+E_s I_{0s}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \\ & -2E_s A_s e_s\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial \bar{w}}{\partial x}\right)\frac{\partial^2 w}{\partial x^2}+G_s J_s\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\end{aligned}\right)\delta(y-n_s d_s)\mathrm{d}x\mathrm{d}y+ \\
& \sum_{n_r=1}^{N_r}\int_0^{2\pi R}\int_0^L\left(\begin{aligned}& E_r A_r\left(\frac{\partial v}{\partial y}+\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial \bar{w}}{\partial y}\right)^2+E_r I_{0r}\left(\frac{\partial^2 w}{\partial y^2}\right)^2 \\ & -2E_r A_r e_r\left(\frac{\partial v}{\partial y}+\frac{w}{R}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial \bar{w}}{\partial y}\right)\frac{\partial^2 w}{\partial y^2}+G_r J_r\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\end{aligned}\right)\delta(x-n_r d_r)\mathrm{d}x\mathrm{d}y- \\
& \frac{1}{2}\int_0^{2\pi R}\int_0^L p_x\left(\frac{\partial w}{\partial x}\right)^2\mathrm{d}x\mathrm{d}y-\frac{1}{2}\sum_{n_s=1}^{N_s}\int_0^{2\pi R}\int_0^L\frac{p_x A_s}{t}\left(\frac{\partial w}{\partial x}\right)^2\delta(y-n_s d_s)\mathrm{d}x\mathrm{d}y
\end{aligned} \tag{13}$$

原型结构参数可由模型结构参数和缩尺因子C_i表示:

$$\begin{aligned}&x_p = C_x x_m, y_p = C_y y_m, z_p = C_z z_m,\\&u_p = C_u u_m, v_p = C_v v_m, w_p = C_w w_m, \overline{w}_p = C_{\overline{w}} \overline{w}_m,\\&L_p = C_L L_m, t_p = C_t t_m, R_p = C_R R_m,\\&e_{sp} = C_{e_s} e_{sm}, e_{rp} = C_{e_r} e_{rm},\\&d_{sp} = C_{d_s} d_{sm}, d_{rp} = C_{d_r} d_{rm},\\&A_{sp} = C_{A_s} A_{sm}, A_{rp} = C_{A_r} A_{rm}, \mu_p = C_\mu \mu_m,\\&E_p = C_E E_m, E_{sp} = C_{E_s} E_{sm}, E_{rp} = C_{E_r} E_{rm},\\&B_p = C_B B_m, D_p = C_D D_m, G_p = C_G G_m,\\&G_{sp} = C_{G_s} G_{sm}, G_{rp} = C_{G_r} G_{rm},\\&J_{sp} = C_{J_s} J_{sm}, J_{rp} = C_{J_r} J_{rm},\\&I_{0sp} = C_{I_{0s}} I_{0sm}, I_{0rp} = C_{I_{0r}} I_{0rm}, p_{xm} = C_{p_x} p_{xm}\end{aligned} \tag{14}$$

当两相似结构的蒙皮完全几何相似时,即 $C_x=C_y=C_u=C_v=C_w=C_R$,并利用 $\delta(ax)=\delta(x)/|a|$[14],将式(14)代入式(13)进行相似转换,可得模型和原型的广义相似条件:

$$\begin{aligned}&C_{d_r} = C_{d_s} = C_{e_r} = C_{e_s} = C_R,\\&C_B C_\mu = C_B = C_G C_t = \frac{C_{E_s} C_{A_s}}{C_R} = \frac{C_{E_r} C_{A_r}}{C_R} =\\&\quad \frac{C_D}{C_R^2} = \frac{C_D C_\mu}{C_R^2} = \frac{C_{E_s} C_{A_s} C_{e_s}}{C_R^2} = \frac{C_{E_r} C_{A_r} C_{e_r}}{C_R^2} =\\&\quad \frac{C_{E_s} C_{I_{0s}}}{C_R^3} = \frac{C_{E_r} C_{I_{0r}}}{C_R^3} = \frac{C_{G_s} C_{J_s}}{C_R^3} = \frac{C_{G_r} C_{J_r}}{C_R^3} =\\&\quad \frac{C_{p_x} C_{A_s}}{C_t C_R} = C_{p_x}\end{aligned} \tag{15}$$

基于式(15),进一步推导得到模型与原型需满足的必要相似条件:

$$C_G \cdot C_t = C_B, C_\mu = 1 \tag{16}$$

$$\begin{aligned}&\frac{C_{E_s} C_{A_s}}{C_R} = \frac{C_{E_r} C_{A_r}}{C_R} = \frac{C_D}{C_R^2} =\\&\quad \frac{C_{E_s} C_{A_s} C_{e_s}}{C_R^2} = \frac{C_{E_r} C_{A_r} C_{e_r}}{C_R^2} =\\&\quad \frac{C_{E_s} C_{I_{0s}}}{C_R^3} = \frac{C_{E_r} C_{I_{0r}}}{C_R^3} =\\&\quad \frac{C_{G_s} C_{J_s}}{C_R^3} = \frac{C_{G_r} C_{J_r}}{C_R^3}\end{aligned} \tag{17}$$

$$C_{A_s} = C_t C_R \tag{18}$$

为了能准确预测原型结构的屈曲特性,原型与模型的屈曲模态相似条件为:

$$C_m = C_n = 1 \tag{19}$$

式中:m 和 n 分别表示圆柱壳纵向和环向的波数.

1.2 纵环离散加筋圆柱壳轴压后屈曲的缩尺原理公式

当加筋圆柱壳受轴向压力 p_x(受压为正)时,根据广义相似条件式(15),可以得到纵环离散加筋圆柱壳轴压后屈曲相似不变量:

$$\frac{C_{\overline{p}_x} \cdot C_R^2}{C_{\text{stiff}}} = 1 \tag{20}$$

式中:$\overline{p}_x$ 表示 x 方向的屈曲临界荷载;C_{stiff} 具体表达式为

$$\begin{aligned}C_{\text{stiff}} &= C_D = C_B \cdot C_R^2 = C_G \cdot C_t \cdot C_R^2 =\\&C_{E_s} \cdot C_{A_s} \cdot C_R = C_{E_r} \cdot C_{A_r} \cdot C_R =\\&\frac{C_{E_s} C_{I_{0s}}}{C_R} = \frac{C_{E_r} C_{I_{0r}}}{C_R} = \frac{C_{G_s} \cdot C_{J_s}}{C_R} = \frac{C_{G_r} \cdot C_{J_r}}{C_R}\end{aligned} \tag{21}$$

基于式(20),推导得纵环离散加筋圆柱壳轴压后屈曲缩尺原理公式为:

$$\left(\overline{p}_x\right)_p = \left(\overline{p}_x\right)_m \frac{C_{\text{stiff}}}{C_R^2} = \left(\overline{p}_x\right)_m \cdot C_{\text{stiff}} \left(\frac{R_m}{R_p}\right)^2 \tag{22}$$

2 算例验证

基于现有文献中的模型,建立相同参数的有限元模型进行屈曲分析,并对比它们的屈曲荷载间的误差,有效验证了建立的含初始缺陷的光滑圆柱壳有限元模型的准确性. 基于 ANSYS 软件建立有限元模型,模型采用 shell181 单元,该单元为四节点六自由度单元,计算时间短,精度高.

2.1 含酒窝缺陷光滑圆柱壳轴压后屈曲验证

采用文献[15]中含酒窝缺陷光滑圆柱壳轴压后屈曲的算例进行有限元模型验证. 结构上端面承受轴压荷载,下端面固支,上端面仅放松轴向位移自由度. 材料和几何参数为:E=72 GPa、μ=0.31、R=0.25 m、L=0.51 m 和 t=0.000 5 m. 在结构高度方向 $L/2$ 位置施加酒窝缺陷,缺陷函数[15]见式(23).

$$\begin{aligned}&\delta_a(s) = \delta_0 e^{-\pi s/\lambda} \cos\left(\frac{\pi s}{\lambda}\right)\\&s = \sqrt{\left(R\theta - R\theta_0\right)^2 + \left(z_1 - z_0\right)^2}\end{aligned} \tag{23}$$

式中:s 为缺陷范围内有限元网格节点到缺陷中心的距离;λ 和 δ_0 分别为酒窝缺陷的直径与中心幅值;θ 和 z_1 分别为光滑圆柱壳的环向和轴向的坐标;θ_0 和 z_0 分别为酒窝缺陷中心的环向和轴向的坐标.

表 1 给出含酒窝缺陷光滑圆柱壳轴压后屈曲临界荷载对比,其中 $p_{\text{cr\_}\alpha}$ 表示文献[15]的临界屈曲荷载,$p_{\text{cr\_FEM}}$ 表示本文计算的临界屈曲荷载. 由结果可知,不同缺陷半径和不同缺陷幅值工况下,光滑圆柱壳轴压屈曲临界荷载与文献[15]结果的误差绝对值均小于8%,表明进行的含酒窝缺陷圆柱壳轴压后屈

曲分析准确、可靠.

表 1 含酒窝缺陷光滑圆柱壳轴压后屈曲临界荷载对比

Tab.1 Comparisons of the buckling results the shells under axial compression with dimple imperfection

缺陷半径/mm	δ_0	p_{cr_α}/(kN·m^{-1})	p_{cr_FEM}/(kN·m^{-1})	误差/%
20	$1t$	47.39	47.46	−0.15
20	$3t$	43.07	46.81	−7.99
20	$5t$	47.39	47.66	−0.57
30	$1t$	40.54	41.23	−1.67
30	$3t$	41.75	40.98	1.88
30	$5t$	42.29	41.91	0.91

2.2 含环向焊缝缺陷光滑圆柱壳轴压后屈曲验证

采用文献[16]中的含环向焊缝缺陷光滑圆柱壳轴压后屈曲的算例进行有限元模型验证.结构两端面承受轴压荷载,上、下端面简支.材料和几何参数为:E=200 GPa、μ=0.3、R=10 m、L=30 m 和 t=0.001 m.在结构高度方向 L/2 位置施加一条环向内陷焊缝,缺陷函数[16]见式(24).

$$\overline{w}(x)=\delta_0 e^{-\pi x/\lambda_0}\left(\cos\frac{\pi x}{\lambda_0}+\sin\frac{\pi x}{\lambda_0}\right) \tag{24}$$

式中:λ_0为焊缝缺陷半波长.

表 2 给出含环向焊缝缺陷光滑圆柱壳轴压后屈曲临界荷载对比,其中 p_{cr_β}表示文献[16]的临界屈曲荷载.

表 2 含环向焊缝缺陷光滑圆柱壳轴压后屈曲临界荷载对比

Tab.2 Comparisons of the buckling results the shells under axial compression with weld depressions

R/t	δ_0	p_{cr_β}/(kN·m^{-1})	p_{cr_FEM}/(kN·m^{-1})	误差/%
100	$1t$	37 039.93(n=13)	37 545.16(n=13)	−1.36
1 000	$0.5t$	522.87(n=22)	540.12(n=22)	−3.30
1 000	$1t$	370.40(n=18)	386.78(n=18)	−4.42
1 000	$1.5t$	305.16(n=14)	328.99(n=14)	−7.83
1 000	$2t$	288.21(n=12)	314.26(n=12)	−9.04
2 000	$1t$	92.60(n=25)	94.91(n=25)	−2.49

从表 2 可以看出,不同径厚比和环向焊缝不同缺陷幅值工况下的光滑圆柱壳,其轴压后屈曲临界荷载与文献[16]结果间的误差绝对值小于 10%.同时,屈曲临界荷载处相应结构的屈曲模态环向波数相同.缺陷幅值小于等于 1.0t 时,误差绝对值在 5% 以内;当缺陷幅值为 1.5t 和 2t 时,误差分别为−7.83% 和−9.04%,误差较大的原因可能是现有的板壳后屈曲分析理论尚不能较好地分析板壳结构缺陷幅值大于 1.0t 的情况.综上所述,开展的含环向焊缝缺陷光滑圆柱壳轴压后屈曲分析具有较好的准确性.

3 纵环离散加筋圆柱壳轴压后屈曲不完全相似分析

为了验证所推导的缩尺原理公式的准确性,首先对纵环加筋圆柱壳缩尺模型和原型进行轴压后屈曲分析.其次,将缩尺模型的计算结果代入缩尺原理公式得到原型的预测结果.最后,将预测原型与原型的荷载位移曲线进行对比,并判断它们的屈曲临界荷载对应的模态是否一致.

纵环加筋圆柱壳原型结构上、下端面简支.蒙皮与加筋肋的材料参数为:E=200 GPa、μ=0.3.加筋肋数量为:N_s=8、N_r=2.蒙皮与加筋肋几何参数分别见表 3 和表 4.

表 3 蒙皮几何参数

Tab.3 Geometry parameters of the skin m

R	L	t
4.85	19.3	0.014 4

表 4 加筋肋几何参数

Tab.4 Geometry parameters of the stiffeners mm

b_{fs}	t_{fs}	h_{ws}	t_{ws}	b_{fr}	t_{fr}	h_{wr}	t_{wr}
254	15.2	152	4.93	254	15.2	152	4.93

因满足全部相似条件极其困难,且完全相似在实际情况中也不适用,故可忽略完全相似的部分非重要相似条件,进行结构的不完全相似研究.通过放松广义相似条件,开展缩尺模型的材料和几何尺寸均发生变化的不完全相似研究.此时,假设模型的蒙皮几何尺寸具有相同的几何缩尺因子,且模型与原型具有相同的边界条件,可得如下相似关系:

$$\begin{aligned}&C_E=k\\&C_R=C_L=C_t=C_{d_s}=C_{d_r}=k_1\end{aligned} \tag{25}$$

将式(25)代入广义相似条件式(15)可得:

$$\begin{aligned}&C_{A_s}=k_1^2, C_{J_s}=k_1^4, C_{z_s}=k_1\\&C_{A_r}=k_1^2, C_{J_r}=k_1^4, C_{z_r}=k_1\end{aligned} \tag{26}$$

缩尺原理公式(22)可简化为:

$$\left(\overline{p}_x\right)_p=\left(\overline{p}_x\right)_m\frac{C_{stiff}}{C_R^2}=\left(\overline{p}_x\right)_m\cdot\frac{C_{stiff}}{k_1^2} \tag{27}$$

此时令 $C_{\mathrm{stiff}} = C_D$，式(27)可进一步简化为：

$$\left(\bar{p}_x\right)_{\mathrm{p}} = \left(\bar{p}_x\right)_{\mathrm{m}} \cdot \frac{kk_1\left(1-\mu_{\mathrm{m}}^2\right)}{\left(1-\mu_{\mathrm{p}}^2\right)} \tag{28}$$

不完全相似缩尺模型的蒙皮几何参数的缩尺因子均取10，同时，缩尺模型的蒙皮和加筋肋的材料分别使用铝(Al)、紫铜(Copper)、黄铜(Brass)和塑料(PVC)，表5列出了缩尺模型的材料参数．根据式(26)求得缩尺模型环向和纵向等效加筋肋的几何尺寸，发现它们的几何缩尺因子与蒙皮的几何缩尺因子相等．

表5　不完全相似缩尺模型材料参数表

Tab.5　Material parameters of the partly similar scale models

材料	E/GPa	μ
Al	68.75	0.3
Copper	124	0.33
Brass	106	0.34
PVC	3.79	0.4

实际工程中的薄壳结构通常存在着一定的初始几何缺陷．其中，酒窝缺陷与焊缝缺陷为典型且不利缺陷．因此，引入不同幅值的酒窝缺陷与环向焊缝缺陷，进行含初始缺陷的纵环离散加筋圆柱壳轴压后屈曲不完全相似分析．

3.1　含酒窝缺陷纵环离散加筋圆柱壳轴压后屈曲不完全相似分析

采用式(23)建立含酒窝缺陷的纵环加筋圆柱壳原型与缩尺模型．其中，缺陷幅值取 δ_0=IF·t，IF代表初始几何缺陷的缺陷因子，在结构环向加筋肋上施加2个酒窝缺陷，结构变形图如图3所示．

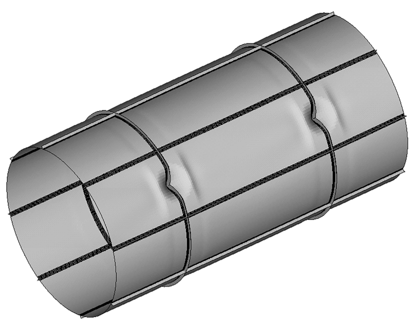
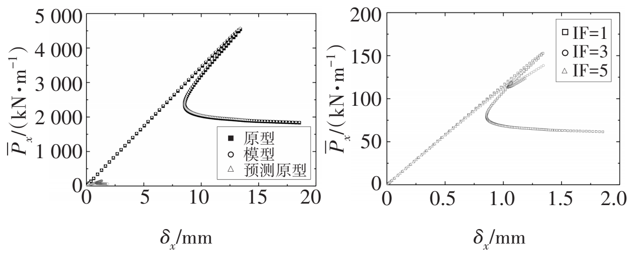


图3　含酒窝缺陷T型纵环加筋圆柱壳结构变形图

Fig.3　Structural deformation diagram of the ring and stringer T-shaped stiffened-cylindrical shell with dimple imperfection

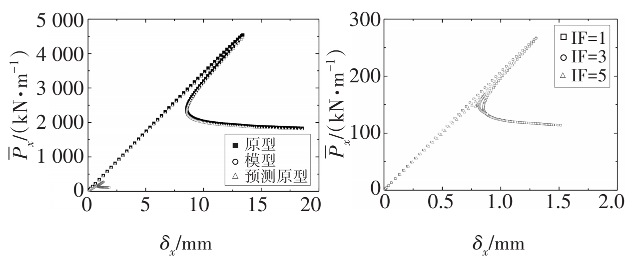
4种不同模型材料的T形纵环加筋圆柱壳轴压后屈曲不完全相似模拟的结果见图4~图7．由图可知，在结构达到屈曲前，其荷载与位移处于近似线性关系；当荷载达到上临界点时，荷载随位移陡然下降，结构进入后屈曲阶段；同时，随着IF值的增大，结构上临界点对应的屈曲荷载逐渐减小．表6为预测原型与原型的上临界屈曲荷载的误差对比．由表6可知，随着模型与原型所取材料泊松比偏差的增大，预测原型与原型的上临界点屈曲荷载的偏差也逐渐增大．IF=1，模型材料为Al时，T型纵环加筋圆柱壳平衡路径上临界点径向位移矢量和云图对比见图8．通过对比上临界点处屈曲模态图可知，上临界点处缩尺模型与其对应原型的屈曲模态均相同．由以上分析可知，基于缩尺原理公式，缩尺模型能较好地预测其对应原型的平衡路径和屈曲模态．



(a)IF=1　(b)模型

图4　模型材料为Al时，T型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

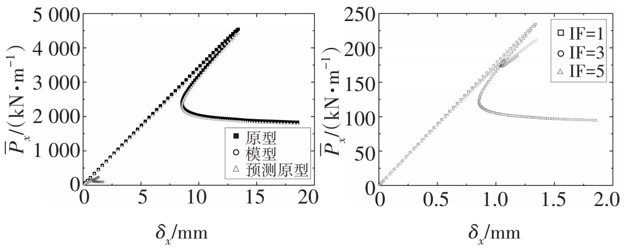
Fig.4　Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is Al



(a)IF=1　(b)模型

图5　模型材料为Copper时，T型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

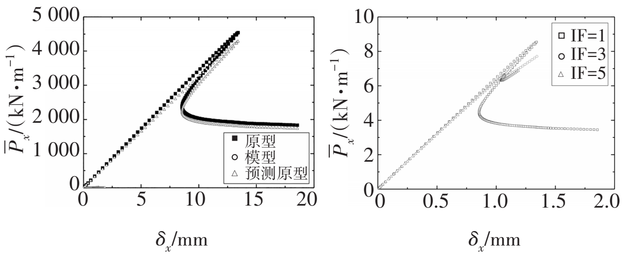
Fig.5　Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is Copper



(a)IF=1 (b)模型

图 6 模型材料为 Brass 时,T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

Fig.6 Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is Brass



(a)IF=1 (b)模型

图 7 模型材料为 PVC 时,T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

Fig.7 Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material of the model is PVC

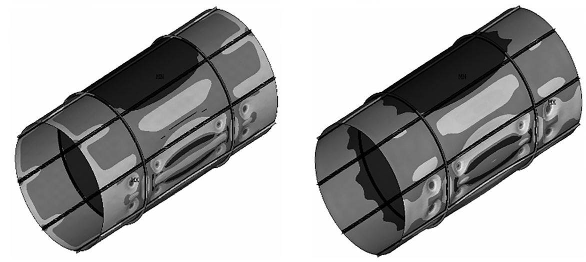
表 6 预测原型与原型上临界屈曲荷载的误差对比

Tab.6 Comparisons of the buckling results between the prototype and the predicted prototypes at the upper critical point of the post-buckling path

材料	误差/%		
	IF=1	IF=3	IF=5
Al	0.81	1.17	1.19
Copper	1.65	1.79	1.68
Brass	2.44	2.57	2.80
PVC	5.63	5.75	5.82

3.2 含环向焊缝缺陷纵环离散加筋圆柱壳轴压后屈曲不完全相似分析

采用式(24)建立含环向焊缝缺陷的纵环加筋圆柱壳原型与缩尺模型,其中缺陷幅值取δ_0=IF·(t+b_f+h_w).在结构每条环向加筋肋翼缘的两端上施加 2 条环向焊缝缺陷,结构变形图见图 9.



(a)模型斜视图 (b)原型斜视图

图 8 IF=1 和模型材料为 Al 时,T 型纵环加筋圆柱壳平衡路径上临界点径向位移矢量和云图对比(放大 10 倍)

Fig.8 The radial displacement contour map at the upper critical point of the post-buckling path for the T-shaped stiffened-cylindrical shell when the material is Al and IF=1(magnified 10 times)

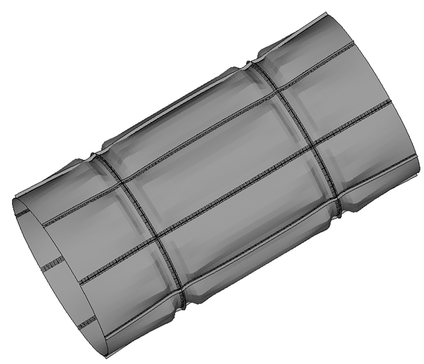
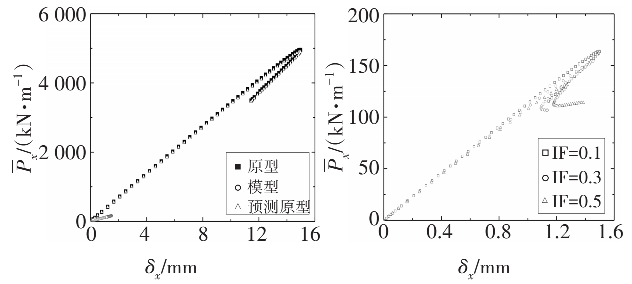


图 9 含环向焊缝缺陷的 T 型纵环加筋圆柱壳结构变形图

Fig.9 Structural deformation diagram of the ring and stringer T-shaped stiffened-cylindrical shell with weld depressions

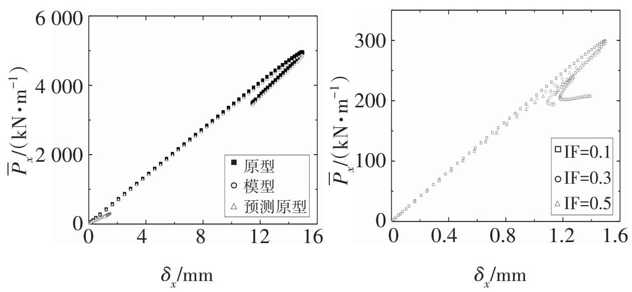
4 种不同模型材料的 T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟的结果见图 10~图 13. 由图可知,结构达到屈曲前,荷载与位移处于近似线性关系;当荷载达到上临界点时,荷载随位移陡然下降,结构进入后屈曲阶段;同时,随着 IF 值的增大,结构上临界点所对应的屈曲荷载逐渐减小. 表 7 为预测原型与原型的上临界屈曲荷载的误差对比. 由表 7 可知,随着模型与原型所取材料的泊松比偏差的增大,预测原型与原型的上临界点屈曲荷载的偏差也逐渐增大. IF=0.1,模型材料为 Al 时,T 型纵环加筋圆柱壳平衡路径上临界点径向位移矢量和云图对比见图 14. 通过对比上临界点处屈曲模态图可知,上临界点处缩尺模型与其对应原型的屈曲模态均相同. 根据以上分析可知,基于缩尺原理公式,缩尺模型能较好地预测其对应原型的平衡路径和屈曲模态.



(a)IF=1　　(b)模型

图 10　模型材料为 Al 时,T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

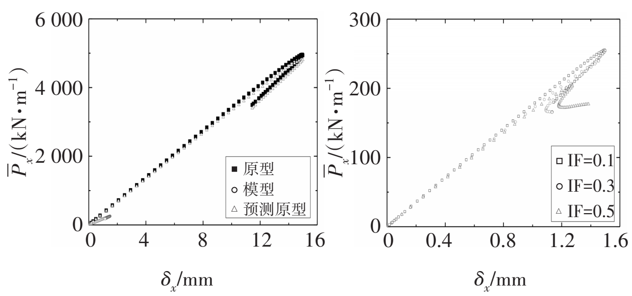
Fig.10　Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is Al



(a)IF=1　　(b)模型

图 11　模型材料为 Copper 时,T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

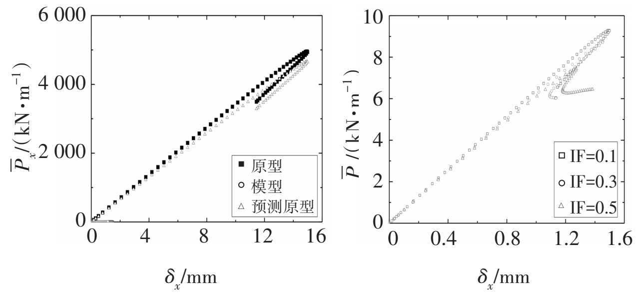
Fig.11　Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is Copper



(a)IF=1　　(b)模型

图 12　模型材料为 Brass 时,T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

Fig.12　Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is Brassa



(a)IF=1　　(b)模型

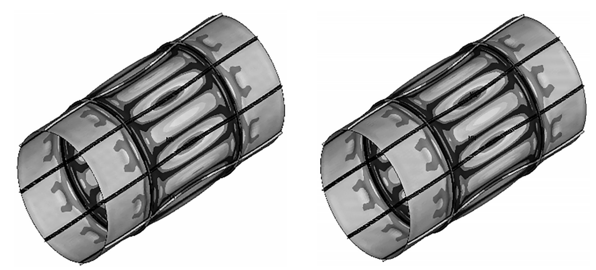
图 13　模型材料为 PVC 时,T 型纵环加筋圆柱壳轴压后屈曲不完全相似模拟

Fig.13　Comparisons between the predicted and original the load vs. end-shortening curves of the T-shaped stiffened-cylindrical shell under axial compression when the material is PVC

表 7　预测原型与原型上临界屈曲荷载的误差对比

Tab.7　Comparisons of the buckling results between the prototype and the predicted prototypes at the upper critical point of the post-buckling path

材料	误差/%		
	IF=0.1	IF=0.3	IF=0.5
Al	0.99	0.71	0.95
Copper	1.85	1.95	2.15
Brass	2.74	2.73	2.94
PVC	5.93	5.89	6.13



(a)模型斜视图　　(b)原型斜视图

图 14　IF=0.1 和模型材料为 Al 时,T 型纵环加筋圆柱壳平衡路径上临界点径向位移矢量和云图对比(放大 10 倍)

Fig.14　The radial displacement contour map at the upper critical point of the post-buckling path for the T-shaped stiffened-cylindrical shell when the material is Al and IF=0.1(magnified 10 times)

4　结　论

本文以纵环离散加筋圆柱壳为研究对象,根据离散加筋理论和能量法推导出结构轴压后屈曲的广

义相似条件与缩尺原理公式;结合轴压后屈曲缩尺原理公式,对含初始缺陷纵环加筋圆柱壳的原型和缩尺模型,进行了结构轴压后屈曲的不完全相似分析.得到如下结论:

1)纵环离散加筋圆柱壳缩尺模型的轴压后屈曲荷载位移曲线,结合结构轴压后屈曲不完全相似缩尺原理公式,能较好预测原型结构轴压后屈曲的结果.随缩尺模型与原型的材料泊松比偏差的增大,由纵环离散加筋圆柱壳不完全相似缩尺模型预测的原型结构荷载位移曲线与原型结构结果的偏差逐渐增大.因此,在利用纵环离散加筋圆柱壳轴压后屈曲不完全相似缩尺模型设计方法和缩尺原理公式进行相似预报时,模型结构与原型结构的材料泊松比应相近.

2)提出的纵环离散加筋圆柱壳轴压后屈曲的缩尺模型设计方法和缩尺原理公式适用于不同几何缺陷形式和缺陷幅值的加筋圆柱壳轴压后屈曲相似预报,并且能比较准确地预报大型加筋圆柱壳轴压后屈曲特性.这为采用离散加筋圆柱壳轴压后屈曲缩尺模型实验预测其对应原型的后屈曲特性提供了参考.

参考文献

[1] 于伟.新型干式煤气柜结构的非线性屈曲与稳定研究[D].重庆:重庆大学,2016:1-14.
YU W. Study on nonlinear buckling and stability of new typical dry gasholder [D]. Chongqing: Chongqing University, 2016: 1-14.(In Chinese)

[2] 杨金花,傅衣铭.具脱层复合材料层合圆柱壳的屈曲分析[J].湖南大学学报(自然科学版),2005,32(5):25-30.
YANG J H, FU Y M. Buckling analysis of composite laminated cylindrical shell with delamination[J]. Journal of Hunan University (Natural Sciences), 2005, 32(5):25-30.(In Chinese)

[3] 向红,傅衣铭,鲁永红.具损伤正交各向异性板的后屈曲分析[J].湖南大学学报(自然科学版),2007,34(9):57-61.
XIANG H, FU Y M, LU Y H. Post-buckling analysis of orthotropic plate considering damage[J]. Journal of Hunan University (Natural Sciences), 2007, 34(9):57-61.(In Chinese)

[4] SINGHATANADGID P, UNGBHAKORN V. Scaling laws for buckling of polar orthotropic annular plates subjected to compressive and torsional loading[J]. Thin-Walled Structures, 2005, 43(7):1115-1129.

[5] HILBURGER M W, HAYNIE W T, LOVEJOY A E, *et al*. Sub-scale and full-scale testing of buckling-critical launch vehicle shell structures[C]// 53rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference. Hawaii: American Institute of Aeronautics and Astronautics, 2012: 1688-1721.

[6] HILBURGER M W, LINDELL M C, WATERS W A, *et al*. Test and analysis of buckling-critical stiffened metallic launch vehicle cylinders[C]//2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference. Florida: American Institute of Aeronautics and Astronautics, 2018: 1697-1726.

[7] 贾冬云,曹平周,姜德进.正多边形气柜立柱结构简化分析方法研究[J].工程力学,2010,27(6):252-256.
JIA D Y, CAO P Z, JIANG D J. Study on simplified analytical method for the column of regular polygon gasholder[J]. Engineering Mechanics,2010,27(6):252-256.(In Chinese)

[8] 贾冬云,曹平周,姜德进,等.大型正多边形气柜壳体的试验研究[J].土木工程学报,2011,44(8):52-58.
JIA D Y, CAO P Z, JIANG D J, *et al*. Experimental study of the shell of a large-scale regular polygon gasholder[J]. China Civil Engineering Journal, 2011, 44(8):52-58.(In Chinese)

[9] UNGBHAKORN V, SINGHATANADGID P. A scaling law for vibration response of laminated doubly curved shallow shells by energy approach[J]. Mechanics of Advanced Materials and Structures,2009,16(5):333-344.

[10] 中国科学院力学研究所固体力学研究室板壳组.加筋圆柱曲板与圆柱壳[M].北京:科学出版社,1983:12-18.
Institute of Mechanics, Chinese Academy of Sciences, Group of Solid Mechanics Research Shell. Stiffened cylindrical panels and cylindrical shells[M]. Beijing: Science Press, 1983: 12-18. (In Chinese)

[11] AMABILI M. Nonlinear vibrations and stability of shells and plates [M]. Cambridge : Cambridge University Press, 2008: 29-32.

[12] FISHER C A. Dynamic buckling of an axially compressed cylindrical shell with discrete rings and stringers[D]. Oklahoma: The University of Oklahoma Graduate College, 1972:66-70.

[13] 陈骥.钢结构稳定理论与设计[M].北京:科学出版社,2001:255-256
CHEN J. Stability of steel structures theory and design[M]. Beijing: Science Press, 2001: 255-256. (In Chinese)

[14] COHEN-TANNOUDJI C, DIU B, LALOE F. Quantum Mechanics, Volume II[M]. New York: Wiley, 1977:1469-1471.

[15] WANG B, HAO P, LI G, *et al*. Determination of realistic worst imperfection for cylindrical shells using surrogate model[J]. Structural & Multidisciplinary Optimization, 2013, 48(4): 777-794.

[16] ROTTER J M, TENG J G. Elastic stability of cylindrical shells with weld depressions[J]. Journal of Structural Engineering, 1989,115(5):1244-1263.